# Sistem linearnih jednačina

Sistem od m jednačina sa n nepoznatih zovemo sistem linearnih jednačina

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \vdots \qquad\quad \vdots \qquad\quad & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned}$$

Sisteme linearnih jednačina možemo riješiti:

a) Gausovom metodom
b) Kramerovom metodom (metoda determinanti)
c) Matričnom metodom
d) Kroneker-Kapelijevom metodom

# Kroneker-Kapelijeva metoda

Neka je dat sistem linearnih jednačina $Ax=b$, gdje su

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m\times n}, \quad x=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad i \quad b=\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}.$$

Matricu $\overline{A}=[A \mid b]$ zovemo proširena matrica.

Teorema (Kroneker-Kapeli):

Sistem ima jedinstveno rješenje ako i samo ako je $\operatorname{rang} A = \operatorname{rang} \overline{A} = n$ ($n$ broj nepoznatih).

Ako je $\operatorname{rang} A = \operatorname{rang} \overline{A} < n$ tada sistem ima $\infty$ mnogo rješenja. ($n - \operatorname{rang} A$ nepoznatih uzima se proizvoljno)

Ako je $\operatorname{rang} A < \operatorname{rang} \overline{A}$ tada sistem nema rješenja.

1. Kroneker-Kapelijevom metodom riješiti sistem jednačina

$$\begin{aligned} 2x + 4y - 5z &= -5 \\ -x - y + z &= 0 \\ 2x + y - z &= 1. \end{aligned}$$

Rj.

$$\overline{A}=[A \mid b]=\left[\begin{array}{ccc|c} 2 & 4 & -5 & -5 \\ -1 & -1 & 1 & 0 \\ 2 & 1 & -1 & 1 \end{array}\right] \overset{I_v \leftrightarrow II_v}{\sim} \left[\begin{array}{ccc|c} -1 & -1 & 1 & 0 \\ 2 & 4 & -5 & -5 \\ 2 & 1 & -1 & 1 \end{array}\right] \underset{III_v + I_v \cdot 2}{\overset{II_v + I_v \cdot 2}{\sim}} \left[\begin{array}{ccc|c} -1 & -1 & 1 & 0 \\ 0 & 2 & -3 & -5 \\ 0 & -1 & 1 & 1 \end{array}\right]$$

$$\overset{II_v \leftrightarrow III_v}{\sim} \left[\begin{array}{ccc|c} -1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 2 & -3 & -5 \end{array}\right] \overset{III_v + II_v \cdot 2}{\sim} \left[\begin{array}{ccc|c} -1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

$\operatorname{rang} A = \operatorname{rang} \overline{A} = 3$

sistem ima jedinstveno rješenje

$$\begin{aligned} -x - y + z &= 0 \\ -y + z &= 1 \\ -z &= -3 \end{aligned}$$

$z = 3$

$$\begin{aligned} -x - y &= -3 \\ -y &= -2 \end{aligned}$$

$y = 2$

$-x - 2 = -3$

$x = 1$

Rješenje sistema je uređena trojka $(1,2,3)$.

2. Kroneker-Kapelijevom metodom rješiti sistem jednačina

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 3x_1 + x_2 - x_3 &= 3 \\ 2x_1 + x_2 \quad &= 2. \end{aligned}$$

Rj. $\bar{A} = [A|b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 3 \\ 2 & 1 & 0 & 2 \end{array}\right] \underset{III_v - I_v \cdot 2}{\overset{II_v - I_v \cdot 3}{\sim}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -4 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \overset{II_v \leftrightarrow III_v}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right]$

$\overset{III_v - II_v \cdot 2}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

rang $A$ = rang $\bar{A} = 2 < 3$

sistem ima $\infty$ mnogo rješenja

3-2 nepoznatih uzimamo proizvoljno

$x_3 = t$

$-x_2 - 2x_3 = 0$

$x_1 + x_2 + x_3 = 1$

$-x_2 - 2t = 0$

$x_2 = -2t$

$x_1 - 2t + t = 1$

$x_1 = t + 1$

Sistem ima beskonačno mnogo rješenja oblika $(t+1, -2t, t)$ gdje je $t \in \mathbb{R}$.

3. Kroneker-Kapelijevom metodom rješiti sistem jednačina

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y + 6z &= 2 \\ 3x + 6y + 9z &= 5. \end{aligned}$$

Rj. $\bar{A} = [A|b] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 3 & 6 & 9 & 5 \end{array}\right] \underset{III_v - I_v \cdot 3}{\overset{II_v - I_v \cdot 2}{\sim}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right]$

rang $A = 1$, rang $\bar{A} = 2$, rang $A <$ rang $\bar{A}$

sistem nema rješenja.

4. Kroneker-Kapelijevom metodom diskutovati rješenja sistema za razne vrijednosti parametra $\lambda$

$$\begin{aligned} \lambda x + y + z &= 1 \\ x + \lambda y + z &= 2 \\ x + y + \lambda z &= -3 \end{aligned}$$

Rj. za $\lambda \in (-\infty, -2) \cup (-2, 1) \cup (1, +\infty)$ sistem ima jedinstveno rješenje $\left(\frac{1}{\lambda-1}, \frac{2}{\lambda-1}, \frac{-3}{\lambda-1}\right)$

za $\lambda = -2$ sistem ima $\infty$ mnogo rješenja $\left(\frac{3t-4}{3}, \frac{3t-5}{3}, t\right), t \in \mathbb{R}$

za $\lambda = 1$ sistem nema rješenja

# Rješiti sistem linearnih jednačina

$$
\begin{aligned}
x - y + z &= 1\\
x - y - z &= 2\\
x + y - z &= 3\\
x + y + z &= 4
\end{aligned}
$$

Rj.-upute:
Rješimo sistem Kronecker-Kapelijevom metodom

$$
\bar{A} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1\\ 1 & -1 & -1 & 2\\ 1 & 1 & -1 & 3\\ 1 & 1 & 1 & 4 \end{array}\right] \sim \dots \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{array}\right]
$$

$\Rightarrow \quad \text{rang}(A) < \text{rang}(\bar{A})$

Dati sistem nema rješenja

# Rješiti sistem linearnih jednačina

$$
\begin{aligned}
x - y + z &= 2 \\
x - y - z &= 3 \\
x + y - z &= 4 \\
x + y + z &= 5
\end{aligned}
$$

Rj.-upute:

Rješimo sistem Kroneker-Kapelijevom metodom

$$\bar{A} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & -1 & -1 & 3 \\ 1 & 1 & -1 & 4 \\ 1 & 1 & 1 & 5 \end{array}\right] \sim \ldots \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \quad rang(A) < rang(\bar{A})$$

Dati sistem nema rješenje.

(#) Riješiti sistem jednačina

$$
\begin{aligned}
x_1 + 2x_2 - 4x_3 + 8x_4 + 12x_5 &= -10 \\
3x_1 + 7x_2 - 15x_3 + 30x_4 + 45x_5 &= -43 \\
-2x_1 - 3x_2 + 6x_3 - 12x_4 - 18x_5 &= 13
\end{aligned}
$$

Rj.-upute:

Sistem ćemo riješiti Kroneker-Kapelijevom metodom

$$\bar{A} = [A \mid b] = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & 8 & 12 & -10 \\ 3 & 7 & -15 & 30 & 45 & -43 \\ -2 & -3 & 6 & -12 & -18 & 13 \end{array}\right] \underset{III_v + I_v \cdot 2}{\overset{II_v + I_v \cdot (-3)}{\sim}}$$

$$\ldots \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & -2 & -3 & 6 \end{array}\right]$$

$$\Rightarrow \left.\begin{array}{l} \text{rang}(A) = 3 \\ \text{rang}(\bar{A}) = 3 \\ \text{broj nepoznatih} = 5 \end{array}\right\} \Rightarrow$$

sistem ima beskonačno mnogo rješenja i dvije promjenjive uzimamo proizvoljno npr. $x_4 = s$, $x_5 = t$
$s, t \in \mathbb{R}$

$$
\begin{aligned}
x_1 &= 4 \\
x_2 &= 5 \\
x_3 &= 6 + 2s + 3t \\
x_4 &= s \\
x_5 &= t
\end{aligned}
$$

$s, t \in \mathbb{R}$

(#) Riješiti sistem jednačina

$$\begin{aligned} x_1 + 2x_2 - 4x_3 - 8x_4 - 12x_5 &= -11 \\ -2x_1 - 3x_2 + 5x_3 + 10x_4 + 15x_5 &= 7 \\ -3x_1 - 5x_2 + 10x_3 + 20x_4 + 30x_5 &= 25 \end{aligned}$$

Rj.-upute:

Sistem ćemo riješiti Kroneker-Kapelijevom metodom

$$\bar{A} = [A \mid b] = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & -8 & -12 & -11 \\ -2 & -3 & 5 & 10 & 15 & 7 \\ -3 & -5 & 10 & 20 & 30 & 25 \end{array}\right] \begin{array}{l} \\ II_v + I_v \cdot 2 \\ \sim \\ III_v + I_v \cdot 3 \end{array}$$

$$\ldots \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 2 & 3 & 7 \end{array}\right]$$

$$\Rightarrow \left.\begin{array}{l} \text{rang}(A) = 3 \\ \text{rang}(\bar{A}) = 3 \\ \text{broj nepoznatih} = 5 \end{array}\right\} \Rightarrow$$

sistem ima beskonačno mnogo rješenja i dvije promjenjive uzimamo proizvoljno npr. $x_4 = s$, $x_5 = t$

$x_1 = 5$

$x_2 = 6$

$x_3 = 7 - 2s - 3t$

$x_4 = s$

$x_5 = t$

$s, t \in \mathbb{R}$

(#) Riješiti sistem jednačina za razne vrijednosti parametra $\lambda \in \mathbb{R}$:

$$\begin{aligned} 2x_1 - x_2 + 3x_3 - 7x_4 &= 15 \\ 6x_1 - 3x_2 + x_3 - 4x_4 &= 7 \\ 4x_1 - 2x_2 + 14x_3 - 31x_4 &= \lambda \end{aligned}$$

Rj.

Rješimo sistem Kroneker-Kapelijevom metodom:

$$\overline{C} = [C \mid b] = \left[\begin{array}{cccc|c} 2 & -1 & 3 & -7 & 15 \\ 6 & -3 & 1 & -4 & 7 \\ 4 & -2 & 14 & -31 & \lambda \end{array}\right] \underset{III_v - I_v \cdot 2}{\overset{II_v - I_v \cdot 3}{\sim}} \left[\begin{array}{cccc|c} 2 & -1 & 3 & -7 & 15 \\ 0 & 0 & -8 & 17 & -38 \\ 0 & 0 & 8 & -17 & \lambda - 30 \end{array}\right]$$

$$\overset{III_v + II_v}{\sim} \left[\begin{array}{cccc|c} 2 & -1 & 3 & -7 & 15 \\ 0 & 0 & -8 & 17 & -38 \\ 0 & 0 & 0 & 0 & \lambda - 68 \end{array}\right]$$

1° $\lambda - 68 \neq 0$, $\lambda \neq 68$

$\operatorname{rang} C = 2$, $\operatorname{rang} \overline{C} = 3$, $\operatorname{rang} C < \operatorname{rang} \overline{C}$

Prema Kroneker-Kapelijevoj teoremi sistem nema rješenje

2° $\lambda - 68 = 0$, $\lambda = 68$

$\operatorname{rang} C = \operatorname{rang} \overline{C} = 2 < 4$ (broj nepoznatih)

Prema Kroneker-Kapelijevoj teoremi dvije promjenjive uzimamo proizvoljno, npr. $x_4 = t$, $x_1 = s$

$$\begin{aligned} 2x_1 - x_2 + 3x_3 - 7x_4 &= 15 \\ -8x_3 + 17x_4 &= -38 \end{aligned}$$

$x_4 = t$

$-8x_3 + 17t = -38$

$-8x_3 = -17t - 38$

$x_3 = \frac{17}{8}t + \frac{38}{8} = \frac{17}{8}t + \frac{19}{4}$

$x_1 = s$

$2s - x_2 + 3\left(\frac{17}{8}t + \frac{38}{8}\right) - 7t = 15$

$x_2 = \frac{51t}{8} + \frac{114}{8} + 2s - 7t - 15$

$x_2 = -\frac{5}{8}t - \frac{6}{8} + 2s$

$x_2 = 2s - \frac{5}{8}t - \frac{3}{4}$

Za $\lambda = 68$ rješenje sistema je

$$\left(s,\ 2s - \frac{5}{8}t - \frac{3}{4},\ \frac{17}{8}t + \frac{19}{4},\ t\right),\quad t, s \in \mathbb{R}$$

(#) Riješiti sistem jednačina za razne vrijednosti parametra $\lambda \in \mathbb{R}$:

$$\begin{aligned} 8x_1 + 12x_2 + 7x_3 + \lambda x_4 &= 9 \\ 6x_1 + 9x_2 + 5x_3 + 6x_4 &= 7 \\ 4x_1 + 6x_2 + 3x_3 + 4x_4 &= 5 \\ 2x_1 + 3x_2 + 2x_3 + 2x_4 &= 2. \end{aligned}$$

Rj: Sistem ćemo rešiti Kroneker-Kapelijevom metodom:

$$\overline{B} = [B \,|\, b] = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \phantom{9} \end{array} \\ \left[\begin{array}{cccc|c} 8 & 12 & 7 & \lambda & 9 \\ 6 & 9 & 5 & 6 & 7 \\ 4 & 6 & 3 & 4 & 5 \\ 2 & 3 & 2 & 2 & 2 \end{array}\right] \end{array} \overset{I_v \leftrightarrow IV_v}{\sim} \left[\begin{array}{cccc|c} 2 & 3 & 2 & 2 & 2 \\ 6 & 9 & 5 & 6 & 7 \\ 4 & 6 & 3 & 4 & 5 \\ 8 & 12 & 7 & \lambda & 9 \end{array}\right] \underset{IV_v - I_v \cdot 4}{\overset{\substack{II_v - I_v \cdot 3 \\ III_v - I_v \cdot 2}}{\sim}}$$

$$\sim \left[\begin{array}{cccc|c} 2 & 3 & 2 & 2 & 2 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & \lambda-8 & 1 \end{array}\right] \underset{IV_v - II_v}{\overset{III_v - II_v}{\sim}} \left[\begin{array}{cccc|c} 2 & 3 & 2 & 2 & 2 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda-8 & 0 \end{array}\right]$$

1° za $\lambda = 8$ imamo $\operatorname{rang} B = \operatorname{rang} \overline{B} = 2 < 4$ pa prema Kroneker-Kapelijevoj teoremi sistem ima $\infty$ mnogo rješenja. Dvije promjenjive uzimamo proizvoljno npr. $x_1 = t$, $x_4 = s$

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 + 2x_4 &= 2 \\ -x_3 + 0x_4 &= 1 \end{aligned}$$

$x_3 = -1$

$2t + 3x_2 - 2 + 2s = 2$

$3x_2 = 4 - 2t - 2s$

$x_2 = \frac{2}{3}(2 - t - s)$

Rješenje sistema (za $\lambda = 8$) je $(t, \frac{2}{3}(2-t-s), -1, s)$ gdje su $s, t \in \mathbb{R}$.

2° za $\lambda \neq 8$ imamo $\operatorname{rang} B = \operatorname{rang} \overline{B} = 3 < 4$ pa prema Kroneker-Kapelijevoj teoremi sistem ima $\infty$ mnogo rješenja. Jednu promjenjivu uzimamo proizvoljno npr. $x_2 = t$.

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 + 2x_4 &= 2 \\ -x_3 &= 1 \\ (\lambda - 8)x_4 &= 0 \end{aligned}$$

$x_4 = 0$

$x_3 = -1$

$2x_1 + 3t - 2 = 2$

$2x_1 = 4 - 3t$

$x_1 = 2 - \frac{3}{2}t$

Rješenje sistema (za $\lambda \neq 8$) je $(2 - \frac{3}{2}t, t, -1, 0)$ gdje su $t \in \mathbb{R}$.

\# Riješiti sistem jednačina za razne vrijednosti parametra $\lambda \in \mathbb{R}$:

$$\begin{aligned} \lambda x_1 - 4x_2 + 9x_3 + 10x_4 &= 11 \\ 2x_1 - x_2 + 3x_3 + 4x_4 &= 5 \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 &= 7 \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 &= 9 \end{aligned}$$

Rj. Sistem ćemo riješiti Kroneker-Kapelijevom metodom:

$$\bar{A} = [A \mid b] = \left[\begin{array}{cccc|c} \lambda & -4 & 9 & 10 & 11 \\ 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \end{array}\right] \underset{I_v \leftrightarrow IV_v}{\sim} \left[\begin{array}{cccc|c} 6 & -3 & 7 & 8 & 9 \\ 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ \lambda & -4 & 9 & 10 & 11 \end{array}\right] \underset{II_v \leftrightarrow I_v}{\sim}$$

$$\sim \left[\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 6 & -3 & 7 & 8 & 9 \\ 4 & -2 & 5 & 6 & 7 \\ \lambda & -4 & 9 & 10 & 11 \end{array}\right] \underset{I_k \leftrightarrow IV_k}{\sim} \begin{array}{c} \begin{array}{cccc} x_4 & x_2 & x_3 & x_1 \end{array} \\ \left[\begin{array}{cccc|c} 4 & -1 & 3 & 2 & 5 \\ 8 & -3 & 7 & 6 & 9 \\ 6 & -2 & 5 & 4 & 7 \\ 10 & -4 & 9 & \lambda & 11 \end{array}\right] \end{array} \underset{I_k \leftrightarrow II_k}{\sim} \begin{array}{c} \begin{array}{cccc} x_2 & x_4 & x_3 & x_1 \end{array} \\ \left[\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ -3 & 8 & 7 & 6 & 9 \\ -2 & 6 & 5 & 4 & 7 \\ -4 & 10 & 9 & \lambda & 11 \end{array}\right] \end{array}$$

$$\underset{\substack{II_v - I_v \cdot 3 \\ III_v - I_v \cdot 2 \\ IV_v - I_v \cdot 4}}{\sim} \left[\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ 0 & -4 & -2 & 0 & -6 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & -6 & -3 & \lambda-8 & -9 \end{array}\right] \underset{II_k \leftrightarrow IV_k}{\sim} \begin{array}{c} \begin{array}{cccc} x_2 & x_1 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} -1 & 2 & 3 & 4 & 5 \\ 0 & 0 & -2 & -4 & -6 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & \lambda-8 & -3 & -6 & -9 \end{array}\right] \end{array} \underset{III_v \leftrightarrow II_v}{\sim} \left[\begin{array}{cccc|c} -1 & 2 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & -2 & -4 & -6 \\ 0 & \lambda-8 & -3 & -6 & -9 \end{array}\right]$$

$$\underset{\substack{III_v - II_v \cdot 2 \\ IV_v - II_v \cdot 3}}{\sim} \begin{array}{c} \begin{array}{cccc} x_2 & x_1 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} -1 & 2 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda-8 & 0 & 0 & 0 \end{array}\right] \end{array}$$

a) Za $\lambda = 8$ imamo $\text{rang}\, A = \text{rang}\, \bar{A} = 2 < 4$ pa prema Kroneker-Kapelijevoj teoremi sistem ima $\infty$ mnogo rješenja.
2. promjenjive uzimamo proizvoljno npr. $x_4 = t$ $x_1 = s$

$$\begin{aligned} -x_3 - 2x_4 &= -3 \\ -x_2 + 2x_1 + 3x_3 + 4x_4 &= 5 \end{aligned}$$

$x_3 = 3 - 2t$

$-x_2 + 2s + 3(3-2t) + 4t = 5$

$x_2 = 2s + 9 - 6t + 4t - 5$

$x_2 = 2s - 2t + 4$

Za $\lambda = 8$ rješenje sistema je $(s, 2s-2t+4, 3-2t, t)$ $t, s \in \mathbb{R}$

b) Za $\lambda \neq 8$ imamo $\text{rang}\, A = \text{rang}\, \bar{A} = 3 < 4$ pa prema Kroneker-Kapelijevom teoremu sistem ima $\infty$ mnogo rješenja.
1. (jednu) promjenjivu uzimamo proizvoljno npr. $x_4 = t$

$$\begin{aligned} (\lambda - 8) x_1 &= 0 \\ -x_3 - 2x_4 &= -3 \\ -x_2 + 2x_1 + 3x_3 + 4x_4 &= 5 \end{aligned}$$

$x_1 = 0$

$x_3 = 3 - 2t$

$-x_2 + 3(3-2t) + 4t = 5$

$x_2 = 9 - 6t + 4t - 5 = -2t + 4$

Za $\lambda \neq 8$ rješenje sistema je $(0, 4-2t, 3-2t, t)$.